Calculus
Exam 3

Name____________________________________

Find the value or values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the given function and interval.

1) $f(x) = x + \frac{50}{x}$, [2, 25].

A) $0, 5\sqrt{2}$ B) $5\sqrt{2}$ C) 2, 25   D) $-5\sqrt{2}, 5\sqrt{2}$

Find the absolute extreme values of the function on the interval.

2) $F(x) = -\frac{3}{x^2}, \frac{1}{2} \le x \le 5$

A) Maximum value of $-\frac{3}{25}$ at $x = 5$; minimum value of -12 at $x = \frac{1}{2}$

B) Maximum value of $-\frac{3}{25}$ at $x = \frac{1}{2}$; minimum value of -12 at $x = -5$

C) Maximum value of $\frac{3}{25}$ at $x = \frac{1}{2}$; minimum value of -12 at $x = -5$

D) Maximum value of $-\frac{3}{25}$ at $x = 5$; minimum value of -12 at $x = -\frac{1}{2}$

3) $f(x) = \sin\left(x + \frac{\pi}{2}\right), 0 \le x \le \frac{7\pi}{4}$

A) Maximum value of 1 at $x = \frac{3}{4}\pi$; minimum value of -1 at $x = \frac{1}{4}\pi$

B) Maximum value of 1 at $x = \frac{1}{4}\pi$; minimum value of -1 at $x = \frac{3}{4}\pi$,

C) Maximum value of 1 at $x = 0$; minimum value of -1 at $x = \pi$

D) Maximum value of 1 at $x = \frac{5}{4}\pi$; minimum value of -1 at $x = \frac{3}{4}\pi$,

4) $f(x) = 4x^{4/3}, -27 \le x \le 1$

A) Maximum value of 4 at $x = 1$; minimum value of 0 at $x = 0$

B) Maximum value of 324 at $x = -27$; minimum value of 4 at $x = 1$

C) Maximum value of 324 at $x = -27$; minimum value of 0 at $x = 0$

D) Maximum value of 81 at $x = -27$; minimum value of 0 at $x = 0$

Find the extreme values of the function and where they occur.

5) $y = \frac{7x}{x^2 + 1}$

A) The maximum value is 0 at $x = 0$.

B) The minimum value is 0 at $x = 0$.

C) The minimum value is $-\frac{7}{2}$ at $x = -1$. The maximum value is $\frac{7}{2}$ at $x = 1$.

D) The minimum value is 0 at $x = 1$. The maximum value is 0 at $x = -1$.

6) $y = x^3 - 3x^2 + 5x - 6$

A) The maximum is 2 at $x = 2$.

B) The maximum is 2 at $x = 1$.

C) None

D) The minimum is 2 at $x = -1$.

Using the derivative of f(x) given below, determine the intervals on which f(x) is increasing or decreasing.

7) $f'(x) = (x - 6)^2(x + 5)$

A) Decreasing on$(-\infty, -5)$ increasing on $(6, \infty)$

B) Decreasing on$(-\infty, -6)$ increasing on $(5, \infty)$

C) Decreasing on$(-\infty, -5)$ increasing on $(-5, 6) \cup (6, \infty)$

D) Decreasing on$(-\infty, -6)$ increasing on $(-6, 5) \cup (5, \infty)$

8) $f'(x) = x^{1/3}(x - 5)$

A) Increasing on $(0, \infty)$

B) Decreasing on $(0, 5)$; increasing on $(5, \infty)$

C) Decreasing on $(-\infty, 0) \cup (5, \infty)$; increasing on $(0, 5)$

D) Decreasing on $(0, 5)$; increasing on $(-\infty, 0) \cup (5, \infty)$

Identify the function's extreme values in the given domain, and say where they are assumed. Tell which of the extreme values, if any, are absolute.

9) $f(x) = -x^5 + 5x^4, \ -\infty < x \leq 5$

A) Local minimum at $x = 0$, $f(0) = 0$; Local maximum at $x = 4$, $f(4) = 256$

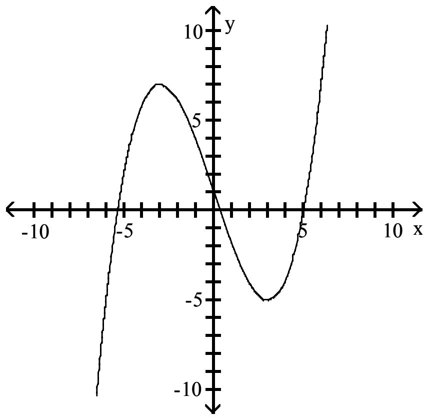
B) Local and absolute minimum at $x = 0$, $f(0) = 0$; Local maximum at $x = 4$, $f(4) = 256$; Local minimum at $x = 5$, $f(5) = 0$

C) Local minimum at $x = 0$, $f(0) = 0$; Local maximum at $x = 4$, $f(4) = 256$; Local minimum at $x = 5$, $f(5) = 0$

D) Local minimum at $x = 0$, $f(0) = 0$; Local and absolute maximum at $x = 4$, $f(4) = 256$; Local minimum at $x = 5$, $f(5) = 0$

Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.

10)



A) Local minimum at x = 3 ; local maximum at x = –3 ; concave up on (–∞, –3) and (3, ∞); concave down on (–3, 3)

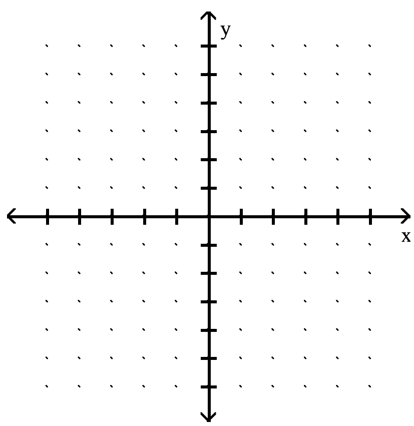
B) Local minimum at x = 3 ; local maximum at x = –3 ; concave down on (–∞, –3) and (3, ∞); concave up on (–3, 3)

C) Local minimum at x = 3 ; local maximum at x = –3 ; concave down on (0, ∞); concave up on (–∞, 0)

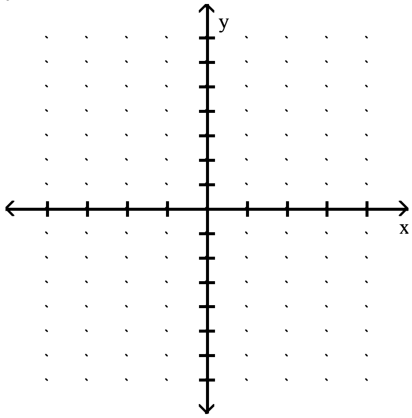
D) Local minimum at x = 3 ; local maximum at x = –3 ; concave up on (0, ∞); concave down on (–∞, 0)

Sketch the graph and show all local extrema and inflection points.

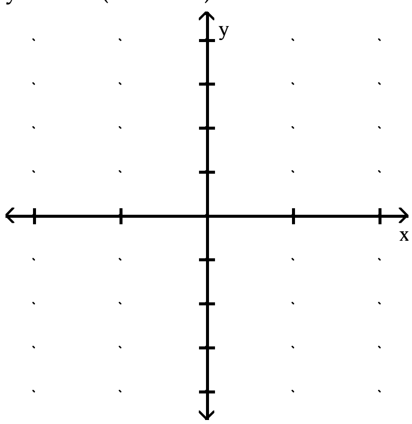
11) $y = \dfrac{16x}{x^2 + 4}$



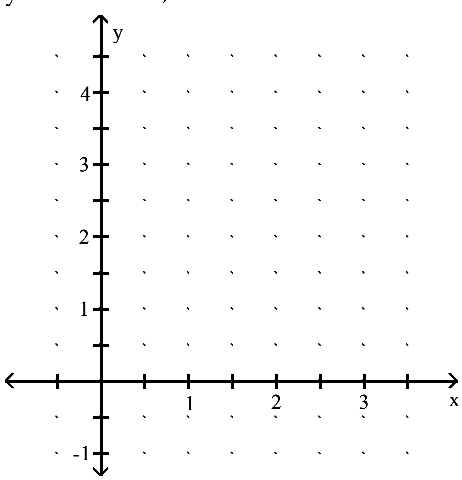
12) $y = 2x^3 - 3x^2 - 12x$



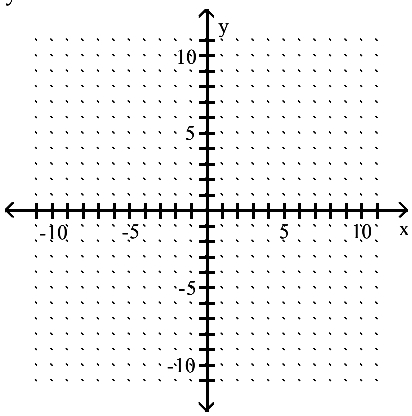
13) $y = x^{1/3}(x^2 - 112)$



14) $y = x + \cos 2x,\ 0 \le x \le \pi$



15) $y = -x^4 + 2x^2 - 8$



Solve the problem.

16) From a thin piece of cardboard 20 in. by 20 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

A) 10 in. by 10 in. by 5 in.; 500 $in.^3$
B) 13.3 in. by 13.3 in. by 6.7 in.; 1185.2 $in.^3$
C) 13.3 in. by 13.3 in. by 3.3 in.; 592.6 $in.^3$
D) 6.7 in. by 6.7 in. by 6.7 in.; 296.3 $in.^3$

17) At noon, ship A was 16 nautical miles due north of ship B. Ship A was sailing south at 16 knots (nautical miles per hour; a nautical mile is 2000 yards) and continued to do so all day. Ship B was sailing east at 6 knots and continued to do so all day. The visibility was 5 nautical miles. Did the ships ever sight each other?

A) No. The closest they ever got to each other was 5.6 nautical miles.
B) Yes. They were within 3 nautical miles of each other.
C) No. The closest they ever got to each other was 6.6 nautical miles.
D) Yes. They were within 4 nautical miles of each other.

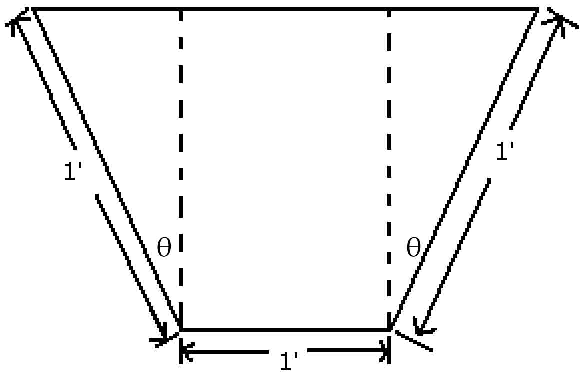
18) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$R(x) = 2x$

$C(x) = 0.001x^2 + 0.6x + 10.$

A) 700 units
B) 2600 units
C) 1300 units
D) 1400 units

19) A trough is to be made with an end of the dimensions shown. The length of the trough is to be 22 feet long. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



A) 8 B) 30° C) 32° D) 52

Find dy.

20) $y = \dfrac{x}{\sqrt{9x + 4}}$

A) $\dfrac{9x - 8}{2(9x + 4)^{3/2}}\, dx$ B) $\dfrac{9x - 8}{2\sqrt{9x + 4}}\, dx$ C) $\dfrac{9x + 8}{2(9x + 4)^{3/2}}\, dx$ D) $\dfrac{9x + 8}{2\sqrt{9x + 4}}\, dx$

The function f(x) changes value when x changes from x_0 to x_0 + dx. Find the approximation error $|\triangle f - df|$. Round your answer, if appropriate.

21) $f(x) = \sqrt{x}$, $x_0 = 4$, $dx = 0.07$

A) −0.00008 B) −0.01758 C) 0.01758 D) 0.00008