

Calculus I  
Review for Final

Name \_\_\_\_\_

**Find the limit, if it exists.**

1)  $\lim_{x \rightarrow 0} \frac{x^3 - 6x + 8}{x - 2}$

2)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

3)  $\lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{x^2 - 4}$

**Find the limit.**

4)  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{2x}$

**Find the limit of the rational function.**

5)  $\lim_{x \rightarrow -\infty} \frac{3x^3 + 4x^2}{x - 5x^2}$

6)  $\lim_{x \rightarrow \infty} \frac{5x^3 + x^2 + 40}{x^4}$

7)  $\lim_{x \rightarrow -\infty} \frac{\cos 5x}{x}$

**Find the slope of the tangent line at the given value of the independent variable.**

8)  $f(x) = 8x + \frac{9}{x}, x = 5$

A)  $\frac{209}{5}$

B)  $\frac{191}{25}$

C)  $\frac{191}{5}$

D)  $\frac{209}{25}$

**Find the derivative.**

9)  $y = \frac{1}{5x^2} + \frac{1}{5x}$

A)  $\frac{2}{5x^3} + \frac{1}{5x^2}$

B)  $-\frac{2}{5x} - \frac{1}{5x^2}$

C)  $-\frac{1}{5x^3} + \frac{1}{5x^2}$

D)  $-\frac{2}{5x^3} - \frac{1}{5x^2}$

Find  $y'$ .

10)  $y = (2x - 3)(3x^3 - x^2 + 1)$

A)  $18x^3 + 33x^2 - 11x + 2$

C)  $6x^3 + 11x^2 - 33x + 2$

B)  $24x^3 - 11x^2 + 33x + 2$

D)  $24x^3 - 33x^2 + 6x + 2$

Find the derivative of the function.

11)  $f(t) = (3 - t)(3 + t^3)^{-1}$

A)  $f'(t) = \frac{2t^3 - 9t^2 - 3}{(3 + t^3)^2}$

C)  $f'(t) = \frac{-4t^3 + 9t^2 - 3}{(3 + t^3)^2}$

B)  $f'(t) = \frac{2t^3 - 9t^2 - 3}{3 + t^3}$

D)  $f'(t) = \frac{-2t^3 + 9t^2 - 3}{(3 + t^3)^2}$

Suppose  $u$  and  $v$  are differentiable functions of  $x$ . Use the given values of the functions and their derivatives to find the value of the indicated derivative.

12)  $u(2) = 6$ ,  $u'(2) = 3$ ,  $v(2) = -2$ ,  $v'(2) = -4$ .

$\frac{d}{dx} \left( \frac{u}{v} \right)$  at  $x = 2$

A)  $-\frac{15}{2}$

B)  $\frac{9}{2}$

C)  $-9$

D)  $\frac{9}{8}$

Find the derivative.

13)  $s = t^6 \tan t - \sqrt{t}$

A)  $\frac{ds}{dt} = t^6 \sec^2 t \tan t + 6t^5 \tan t - \frac{1}{2\sqrt{t}}$

C)  $\frac{ds}{dt} = t^6 \sec^2 t + 6t^5 \tan t - \frac{1}{2\sqrt{t}}$

B)  $\frac{ds}{dt} = -t^6 \sec^2 t + 6t^5 \tan t + \frac{1}{2\sqrt{t}}$

D)  $\frac{ds}{dt} = 6t^5 \sec^2 t - \frac{1}{2\sqrt{t}}$

Find the derivative of the function.

14)  $q = \cos(\sqrt{8t + 12})$

A)  $\frac{dq}{dt} = -\sin\left(\frac{4}{\sqrt{8t + 12}}\right)$

C)  $\frac{dq}{dt} = -\frac{4}{\sqrt{8t + 12}} \sin(\sqrt{8t + 12})$

B)  $\frac{dq}{dt} = -\frac{1}{2\sqrt{8t + 12}} \sin(\sqrt{8t + 12})$

D)  $\frac{dq}{dt} = -\sin(\sqrt{8t + 12})$

15)  $h(\theta) = \sqrt{5 + \sin(12\theta)}$

A)  $h'(\theta) = \frac{6 \cos(12\theta)}{\sqrt{5 + \sin(12\theta)}}$

C)  $h'(\theta) = \frac{1}{2\sqrt{5 + \sin(12\theta)}}$

B)  $h'(\theta) = \frac{\cos(12\theta)}{(5 + \sin(12\theta))^{3/2}}$

D)  $h'(\theta) = \frac{\cos(12\theta)}{2\sqrt{5 + \sin(12\theta)}}$

Use implicit differentiation to find  $dy/dx$ .

16)  $2xy - y^2 = 1$

A)  $\frac{y}{y - x}$

B)  $\frac{x}{x - y}$

C)  $\frac{y}{x - y}$

D)  $\frac{x}{y - x}$

17)  $x^4 = \cot y$

A)  $\frac{4x^3}{\csc^2 y}$

B)  $\frac{\csc^2 y}{4x^3}$

C)  $-\frac{4x^3}{\csc y \cot y}$

D)  $-\frac{4x^3}{\csc^2 y}$

Find the value or values of  $c$  that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the given function and interval.

18)  $f(x) = x + \frac{50}{x}$ ,  $[2, 25]$ .

A) 2, 25

B)  $-5\sqrt{2}$ ,  $5\sqrt{2}$

C)  $5\sqrt{2}$

D) 0,  $5\sqrt{2}$

Find the absolute extreme values of the function on the interval.

19)  $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ ,  $0 \leq x \leq \frac{7\pi}{4}$

A) Maximum value of 1 at  $x = \frac{5}{4}\pi$ ; minimum value of -1 at  $x = \frac{3}{4}\pi$ ,

B) Maximum value of 1 at  $x = \frac{1}{4}\pi$ ; minimum value of -1 at  $x = \frac{3}{4}\pi$ ,

C) Maximum value of 1 at  $x = 0$ ; minimum value of -1 at  $x = \pi$

D) Maximum value of 1 at  $x = \frac{3}{4}\pi$ ; minimum value of -1 at  $x = \frac{1}{4}\pi$

20)  $f(x) = 4x^{4/3}$ ,  $-27 \leq x \leq 1$

A) Maximum value of 4 at  $x = 1$ ; minimum value of 0 at  $x = 0$

B) Maximum value of 324 at  $x = -27$ ; minimum value of 4 at  $x = 1$

C) Maximum value of 81 at  $x = -27$ ; minimum value of 0 at  $x = 0$

D) Maximum value of 324 at  $x = -27$ ; minimum value of 0 at  $x = 0$

Find the extreme values of the function and where they occur.

21)  $y = \frac{7x}{x^2 + 1}$

A) The minimum value is 0 at  $x = 1$ . The maximum value is 0 at  $x = -1$ .

B) The minimum value is 0 at  $x = 0$ .

C) The maximum value is 0 at  $x = 0$ .

D) The minimum value is  $-\frac{7}{2}$  at  $x = -1$ . The maximum value is  $\frac{7}{2}$  at  $x = 1$ .

Using the derivative of  $f(x)$  given below, determine the intervals on which  $f(x)$  is increasing or decreasing.

22)  $f'(x) = (x - 6)^2(x + 5)$

A) Decreasing on  $(-\infty, -6)$  increasing on  $(-6, 5) \cup (5, \infty)$

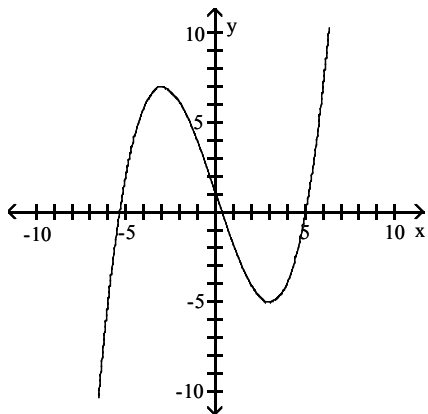
B) Decreasing on  $(-\infty, -5)$  increasing on  $(6, \infty)$

C) Decreasing on  $(-\infty, -5)$  increasing on  $(-5, 6) \cup (6, \infty)$

D) Decreasing on  $(-\infty, -6)$  increasing on  $(5, \infty)$

Use the graph of the function  $f(x)$  to locate the local extrema and identify the intervals where the function is concave up and concave down.

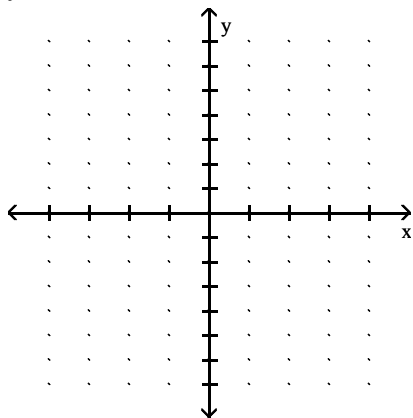
23)



- A) Local minimum at  $x = 3$  ; local maximum at  $x = -3$  ; concave down on  $(-\infty, -3)$  and  $(3, \infty)$ ; concave up on  $(-3, 3)$
- B) Local minimum at  $x = 3$  ; local maximum at  $x = -3$  ; concave up on  $(-\infty, -3)$  and  $(3, \infty)$ ; concave down on  $(-3, 3)$
- C) Local minimum at  $x = 3$  ; local maximum at  $x = -3$  ; concave down on  $(0, \infty)$ ; concave up on  $(-\infty, 0)$
- D) Local minimum at  $x = 3$  ; local maximum at  $x = -3$  ; concave up on  $(0, \infty)$ ; concave down on  $(-\infty, 0)$

Sketch the graph and show all local extrema and inflection points.

24)  $y = 2x^3 - 3x^2 - 12x$



Solve the problem.

25) Suppose that  $f$  is continuous and that  $\int_{-3}^3 f(z) dz = 0$  and  $\int_{-3}^5 f(z) dz = 4$ . Find  $-\int_3^5 5f(x) dx$ .

- A) -4
- B) -5
- C) -20
- D) 20

Evaluate the integral.

26)  $\int_0^4 3\sqrt{x} dx$

- A) 16
- B) 24
- C) 6
- D) 36

$$27) \int_4^9 \frac{t^2+1}{\sqrt{t}} dt$$

A)  $\frac{432}{5}$

B) 212

C)  $\frac{472}{5}$

D)  $\frac{447}{5}$

$$28) \int_{-\pi/4}^{3\pi/4} 9 \sec \theta \tan \theta d\theta$$

A)  $-9\sqrt{2}$

B)  $9\sqrt{2}$

C) 0

D)  $-18\sqrt{2}$

**Find the derivative.**

$$29) y = \int_0^{\tan x} \sqrt{t} dt$$

A)  $\sec^2 x \sqrt{\tan x}$

B)  $\sqrt{\tan x}$

C)  $\sec x \tan^{3/2} x$

D)  $\frac{2}{3} \tan^{3/2} x$

**Evaluate the integral.**

$$30) \int 8x^2 \sqrt[4]{8+2x^3} dx$$

A)  $-\frac{16}{3}(8+2x^3)^{-3/4} + C$

B)  $\frac{16}{15}(8+2x^3)^{5/4} + C$

C)  $\frac{32}{5}(8+2x^3)^{5/4} + C$

D)  $8(8+2x^3)^{5/4} + C$

$$31) \int \csc^2(3\theta+4) d\theta$$

A)  $-\frac{1}{3} \cot(3\theta+4) + C$

B)  $-\cot(3\theta+4) + C$

C)  $3 \cot(3\theta+4) + C$

D)  $6 \csc(3\theta+4) \cot(3\theta+4) + C$

**Find the integral.**

$$32) \int \frac{x}{(1-x)^9} dx$$

A)  $-\frac{1}{7} \left\{ \frac{1}{(1-x)^7} \right\} + \frac{1}{8} \left\{ \frac{1}{(1-x)^8} \right\} + C$

B)  $\frac{1}{8} \left\{ \frac{1}{(1-x)^7} \right\} - \frac{1}{7} \left\{ \frac{1}{(1-x)^8} \right\} + C$

C)  $-\frac{1}{8} \left\{ \frac{1}{(1-x)^7} \right\} + \frac{1}{7} \left\{ \frac{1}{(1-x)^8} \right\} + C$

D)  $\frac{1}{7} \left\{ \frac{1}{(1-x)^7} \right\} - \frac{1}{8} \left\{ \frac{1}{(1-x)^8} \right\} + C$

Use the substitution formula to evaluate the integral.

$$33) \int_0^1 (8y^2 - y + 1)^{-1/3} (64y - 4) dy$$

A) 18

B)  $\frac{9}{2}$

C) 8

D) 24

$$34) \int_{\pi/3}^{2\pi} 4 \cos^3 x \sin x dx$$

A)  $\frac{15}{16}$

B)  $-\frac{15}{4}$

C)  $-\frac{32769}{524288}$

D)  $-\frac{15}{16}$

Evaluate the definite integral.

$$35) \int_0^2 \frac{x}{(2x^2 + 2)^2} dx$$

A) 0.1000

B) -0.1250

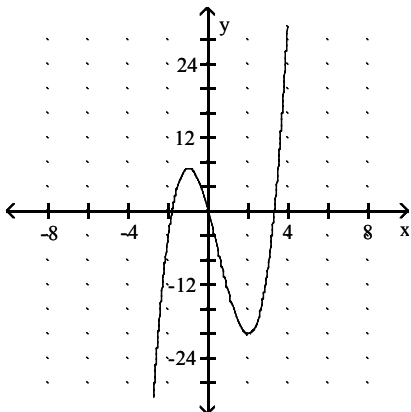
C) 0.4000

D) 0.0060

# Answer Key

Testname: CAL REVIEW FINAL

- 1) -4
- 2)  $\frac{1}{2}$
- 3)  $\frac{5}{2}$
- 4) 0
- 5)  $\infty$
- 6) 0
- 7) 0
- 8) B
- 9) D
- 10) D
- 11) A
- 12) B
- 13) C
- 14) C
- 15) A
- 16) A
- 17) D
- 18) C
- 19) C
- 20) D
- 21) D
- 22) C
- 23) D
- 24) Local max:  $(-1, 7)$ , min:  $(2, -20)$   
Inflection point:  $\left(\frac{1}{2}, -\frac{13}{2}\right)$



- 25) C
- 26) A
- 27) A
- 28) D
- 29) A
- 30) B
- 31) A
- 32) A

## Answer Key

Testname: CAL REVIEW FINAL

33) A

34) D

35) A