

MATH 1325 – CALCULUS WITH APPLICATIONS
FINAL REVIEW PROBLEMS
062

Answer the question.

1) If $f(x) = 2x^2 - 1$, find the following:

a) $\lim_{x \rightarrow 3} f(x)$

b) the average rate of change as x changes from 2 to 5

2) Let $C(x) = 5x + 200$ describe the cost of producing x items.

a) Find the cost of producing 50 items.

b) What is the marginal cost?

c) What is the fixed cost?

d) If this function were graphed, what would be the slope? What would be the y -intercept?

3) Find the slope of the tangent line to $f(x) = 3x^2 - 2x$ when $x = -1$.

What is the equation of the tangent line to the curve when $x = -1$?

Evaluate the limit, if it exists.

4) a) $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x - 1}$

b) $\lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x - 1}$

c) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

Answer the question.

5) An object falls freely a distance of $s(t) = 16t^2$ feet in t seconds after it is dropped.

What is its average velocity between 1 and 4 seconds? What is its instantaneous velocity after 4 seconds?

Find the indicated derivatives.

6) Find y' and y'' for $y = 2x + \frac{1}{x} + 4\sqrt{x}$

Find the derivative using the definition of the derivative.

7) $y = 3x^2 + 5x$

Find the derivative of each function.

8) a) $y = (2x^2 - 5)(x^3 - 1)$

b) $f(x) = \frac{3x + 2}{x^2 - x}$

c) $y = (3x^6 - 9x^3)^5$

d) $y = e^{5x}$

e) $y = \sqrt{2x^4 - 5x^2}$

f) $y = \ln x^2$

g) $f(x) = x^2 \ln x$

h) $f(x) = 3x\sqrt{x+6}$

i) $y = 2x(2x^2 + 1)^5$

j) $y = \frac{\ln(2x)}{x}$

k) $y = 2xe^{x^2}$

l) $y = \frac{x^2 - x^{-2}}{x}$

m) $y = x^2(\sqrt{x} - 1)$

Answer the question.

9) Let $y = 2x^3 - 3x^2$.

- Find the relative extrema.
- Find the intervals where the function is increasing and decreasing.
- Find the point(s) of inflection.
- Find the intervals where the graph is concave up and concave down.
- Find the x-intercept(s).
- Find the y-intercept.
- Find the absolute extrema on $[0, 3]$.
- Sketch the graph.

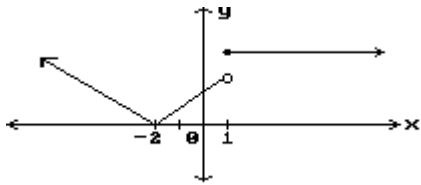
10) The profit when x items are produced is given by $P(x) = 6x - 200 - 0.03x^2$.

- Find the marginal profit.
- For what value of x is the profit a maximum?
- Why is the profit a maximum, not a minimum, at this point?
- What is the maximum profit?

Answer the questions for the given graphs.

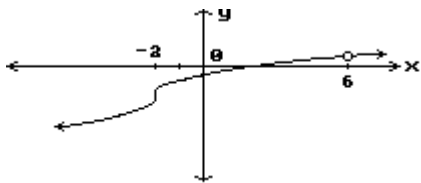
- Evaluate the limit.
- State where the function is discontinuous, if at all.
- State where the function is not differentiable, if at all.

11)



$$\lim_{x \rightarrow -2} f(x)$$

12)



$$\lim_{x \rightarrow 6} f(x)$$

Solve the problem.

- A farmer wishes to enclose a rectangular grazing region with one side along a river. If no fence is needed along the river, find the dimensions of the grazing region of maximum area that can be made with 1200 meters of fence.
- The marginal revenue is given by $R'(x) = 2x - \frac{2}{3}$. The revenue obtained by selling one item is \$10. Find an equation for the revenue. What is the revenue obtained from selling 4 items?

Find antiderivatives of the following functions.

- $x^4 + \sqrt[3]{x} + \frac{3}{x}$
- e^{-3x}
- $x(x^2 + 7)^4$
- $\frac{x}{x^2 + 1}$
- $\frac{x^2 - 3x}{x}$

Find the areas. (Graph the region(s) first.)

- Between the x-axis and the curve $y = x^2 + 1$ from $x = 0$ to $x = 4$
- Between the x-axis and the curve $y = -x^4 - 2$ from $x = -2$ to $x = 0$
- Between the x-axis and the graph of $f(x) = 1 - x^2$ from $x = -1$ to $x = 2$

Find the area.

17) Find the area between the graphs of $f(x) = x^2$ and $g(x) = 2x + 3$. Graph the functions first.

Evaluate the definite integral.

18) Evaluate the following definite integrals:

a) $\int_1^5 \sqrt{2r - 1} \, dr$

b) $\int_{-1}^1 \frac{2}{(2x - 3)^2} dx$

c) $\int_1^5 (3x^2 - 2x) \, dx$

d) $\int_{-1}^3 \frac{x \, dx}{\sqrt{7 + x^2}}$

e) $\int_e^{e^2} \frac{2}{x} \, dx$

Graph the functions.

19) Find the x-intercept(s), y-intercept, vertex if applicable, asymptotes if applicable, relative extrema, and inflection point(s).

a) $y = (x + 2)^2 - 1$

b) $y = \frac{x}{x - 1}$

Find the indicated partial derivatives.

20) a) f_x and f_y for $f(x,y) = e^x \ln y$

b) f_y for $f(x,y) = (xy^2 + 1)^4$

c) f_{xx} and f_{xy} for $f(x,y) = x^4 + 2x^2y^2 + x^3y + y^4$

Find a general solution for the differential equation.

21) a) $\frac{dy}{dx} = xe^{x^2}$

b) $\frac{dy}{dx} = \sqrt{x} + \frac{3}{x}$

Find the particular solution for the differential equation.

22) $\frac{dy}{dx} - 2 = 2x^3 - 3x^2$; $y = 0$ when $x = 2$

Solve the problem.

23) Find the total change in the cost as the production changes from $x = 50$ to $x = 100$ units if $C'(x) = 2 - 0.01x$.

Answer the question.

24) Find the relative extrema for $f(x) = (x + 2)(x - 1)^2$

25) Find the intervals of concavity for $f(x) = \frac{x - 1}{x + 1}$.

Answer Key

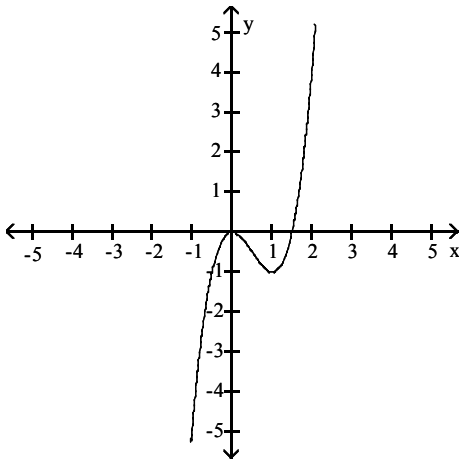
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- 1) a) 17
b) 14
- 2) a) \$450
b) 5
c) \$200
d) $m=5$; $b=200$
- 3) -8; $y = -8x - 3$
- 4) a) 6
b) does not exist
c) $\frac{1}{4}$
- 5) 80 ft/sec; 128 ft/sec
- 6) $y' = 2 - \frac{1}{x^2} + \frac{2}{\sqrt{x}}$
 $y'' = \frac{2}{x^3} - \frac{1}{x\sqrt{x}}$
- 7) $y' = 6x + 5$
- 8) a) $10x^4 - 15x^2 - 4x$
b) $\frac{-3x^2 - 4x + 2}{(x^2 - x)^2}$
c) $(90x^5 - 135x^2)(3x^6 - 9x^3)^4$
d) $5e^{5x}$
e) $\frac{4x^3 - 5x}{\sqrt{2x^4 - 5x^2}}$
f) $\frac{2}{x}$
g) $2x \ln x + x$
h) $\frac{9x + 36}{2\sqrt{x + 6}}$
i) $2(22x^2 + 1)(2x^2 + 1)^4$
j) $\frac{1 - \ln(2x)}{x^2}$
k) $2e^{x^2}(2x^2 + 1)$
l) $1 + \frac{3}{x^4}$
m) $\frac{5}{2}x\sqrt{x} - 2x$

Answer Key

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- 9) a) maximum @ (0,0) ; minimum @ (1, -1)
b) increasing on $(-\infty, 0)$ and $(1, \infty)$; decreasing on $(0, 1)$
c) $(\frac{1}{2}, -\frac{1}{2})$
d) concave up on $(\frac{1}{2}, \infty)$; concave down on $(-\infty, \frac{1}{2})$
e) $0, \frac{3}{2}$
f) 0
g) absolute maximum = 27; absolute minimum = -1
h)



- 10) a) $P'(x) = 6 - 0.06x$
b) $x = 100$
c) $P''(100) = -0.06$. By the 2nd derivative test, profit is a maximum at this point.
d) $P(100) = 100$
- 11) a) 0
b) $x = 1$
c) $x = -2, x = 1$
- 12) a) .5
b) $x = 6$
c) $x = -2, x = 6$
- 13) 300m by 600m
- 14) $R(x) = x^2 - \frac{2}{3}x + 9\frac{2}{3}$
 $R(4) = \$23$

Answer Key

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15) a) $\frac{1}{5}x^5 + \frac{3}{4}x^{\frac{4}{3}} + 3\ln|x| + C$

b) $-\frac{1}{3}e^{-3x} + C$

c) $\frac{1}{10}(x^2 + 7)^5 + C$

d) $\frac{1}{2}\ln(x^2 + 1) + C$

e) $\frac{1}{2}x^2 - 3x + C$

16) a) $25\frac{1}{3}$ square units

b) $10\frac{2}{5}$ square units

c) $2\frac{2}{3}$ square units

17) $10\frac{2}{3}$ square units

18) a) $\frac{26}{3}$

b) 0.8

c) 100

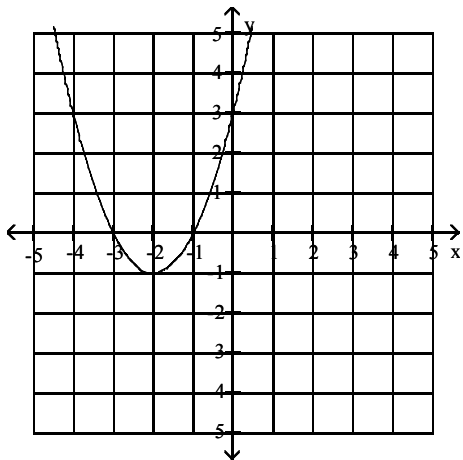
d) $4 - 2\sqrt{2}$

e) 2

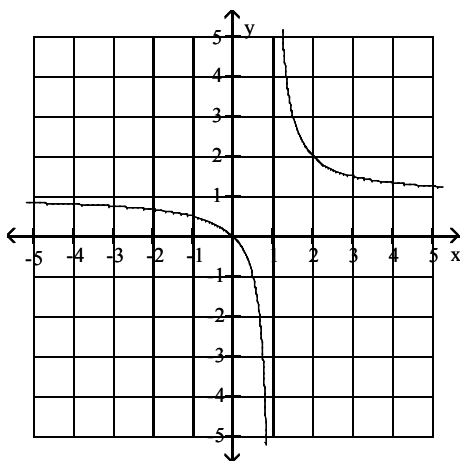
Answer Key

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19) a)



b) VA: $x=1$ HA: $y=1$



20) a) $f_x = e^x \ln y$

$$f_y = \frac{e^x}{y}$$

b) $f_y = 8xy(xy^2 + 1)^3$

c) $f_{xx} = 12x^2 + 4y^2 + 6xy$

$$f_{xy} = 8xy + 3x^2$$

21) a) $y = \frac{1}{2}e^{x^2} + C$

b) $y = \frac{2}{3}x^{\frac{3}{2}} + 3\ln|x| + C$

22) $y = \frac{1}{2}x^4 - x^3 + 2x - 4$

23) \$62.50

24) relative maximum @ (-1, 4)

relative minimum @ (1, 0)

Answer Key

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- 25) concave up on $(-\infty, -1)$
concave down on $(-1, \infty)$